

to be solved by fairly simple methods. There is, however, another class of problems of great practical importance, in which it is not allowable to neglect these quadratic terms, and towards the solution of such problems theory has as yet made little progress.

When a sphere is constrained to move along a horizontal straight line, but is otherwise free, it is well known that if the surrounding liquid is supposed to be frictionless, its only effect is to increase the inertia of the sphere by half the mass of the liquid displaced. The sphere accordingly requires a larger impulsive force to start it than if the liquid were absent, but when once started it continues to move with its velocity of projection. But when the sphere is surrounded by an *actual* liquid, its velocity gradually diminishes until it ultimately comes to rest; and this fact shows very forcibly the necessity of taking the viscosity of the liquid into account in problems of this character. I obtained a few years ago a mathematical solution, which shows that this effect must necessarily be produced by a *viscous* liquid, but the solution is an imperfect one, as mathematical difficulties compelled me to disregard the quadratic terms.

It is always a great advantage when the solution of a mathematical problem can be made to depend upon a *single* function which satisfies a partial differential equation and certain boundary conditions. This is always the case when a solid of revolution moves along its axis in a viscous liquid which is initially at rest, or has an independent motion which is symmetrical with respect to the axis. In this particular class of problems, the motion can be expressed by means of Stokes's current function in the following manner:—Let  $z$  be measured along, and  $r$  perpendicularly to the fixed straight line with which the axis coincides during the motion; let  $w$  and  $u$  be the velocities of the liquid in these directions; then:—

$$u = -\frac{1}{r} \frac{d\psi}{dz}, \quad w = \frac{1}{r} \frac{d\psi}{dr},$$

$$\left( \nu D - \frac{d}{dt} - u \frac{d}{dr} - w \frac{d}{dz} + \frac{2u}{r} \right) D\psi = 0,$$

where

$$D = \frac{d^2}{dz^2} + \frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr},$$

and  $\nu$  is the kinematic coefficient of viscosity.

So far as I am aware, no serious attempt has been made to obtain a solution of this equation in a suitable form, even when the solid is a sphere. The equation is well worthy of the attentive consideration of mathematicians; and although it is an intractable one, it must be recollected that a general solution is not required, but only a particular one which is suitable in the case of a sphere. It will be quite time enough to consider the possibility of obtaining solutions of a more general character, when the appropriate one in the case of a sphere has been discovered. It is also important to recollect that in most problems which are of practical interest,  $\nu$  is a small quantity (about '014 in C.G.S. units for water), and consequently an approximate solution in which  $\nu$  is supposed to be small would meet the exigencies of the case.

When a solid body is moving through a liquid, one of the boundary conditions is that the normal velocity of the solid must be equal to the component along the normal of the velocity of the liquid in contact with it. If the liquid is frictionless, this condition is the only one which has to be satisfied; but when the liquid is viscous, a further question arises as to the law which expresses the effect of the tangential stress exerted by the liquid upon the solid. When the motion is very slow (as in the case of problems relating to small oscillations) the experimental evidence is in favour of the hypothesis of *no slipping*; but when the velocity is considerable, the experimental evidence is not so satisfactory. The partial slipping which takes place under these circumstances must depend partly upon the nature of the liquid, and partly upon that of the surface in contact with it; and the tangential stress to which it gives rise is probably approximately proportional to the square of the relative velocity.

When the motion is symmetrical with respect to an axis, the stresses due to viscosity can be calculated as soon as the value of  $\psi$  is known, the resistance which the liquid exerts on the solid can be found, and the equation of motion written down and integrated. This process is, however, an exceedingly tedious one; but it can always be dispensed with in the case of a single solid by employing the principle of momentum. When the

motion is not symmetrical with respect to an axis, it cannot be expressed in terms of  $\psi$ ; but if the velocities of the liquid can be found from the hydrodynamical equations, the components of the linear and angular momenta of the liquid can be calculated, and by applying the principle of momentum to the compound system composed of the solid and the surrounding liquid, the equations of motion of the former can be obtained. Since the momentum of the system is obviously a function of the six co-ordinates of the solid, this principle furnishes a sufficient number of equations for the determination of the motion.

When there is more than one solid, the principle of momentum is insufficient to determine the motion; but if the velocities of the liquid in the neighbourhood of each solid could be found, the force and couple constituents of the resistance could be calculated, and the equations of motion of each solid written down. Lagrange's equations in their ordinary form cannot be employed, as viscous motion involves a conversion of energy into heat; but problems which can be solved by an indirect method can usually be solved by a direct one, and I feel confident that equations analogous to Lagrange's equations exist, by means of which the motion of a number of solids in a viscous liquid can be found without going through the above-mentioned process. A form of Lagrange's equations has already been discovered, which is applicable when the viscous forces depend upon a dissipation function which is expressible as a homogeneous quadratic function of the velocities; and the circumstance that a dissipation function also exists in the hydrodynamical theory, although it is expressed in a different form, furnishes additional grounds for believing in the existence of equations of this character. The discovery of such equations would constitute an important advance in the theory of viscous liquids.

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#### SCIENCE IN THE PUBLIC SCHOOLS AND IN THE SCIENTIFIC BRANCHES OF THE ARMY.

ON Friday last Mr. Campbell Bannerman received a deputation on this subject in his room in the House of Commons. There were present Sir Henry Roscoe, the Head Master of Rugby School, the Principal of Cheltenham College, the Head Master of Clifton College, Sir B. Samuelson, Prof. Jelf, and Mr. Shenstone. Lord Playfair, Sir John Lubbock, and Sir Henry Howorth would also have been present, but they were prevented by other engagements. The following is a brief account of the proceedings:—

Sir Henry Roscoe, in introducing the deputation, said that he had introduced a deputation on this subject to Mr. Stanhope about five years ago, and that if the suggestions then made had been adopted the present deputation would not have been necessary. After some remarks which showed the injustice of the present system to the more scientific lads, he pointed out several methods by which this injustice might be removed.

The Head Master of Rugby, Dr. Percival, expressed his strong feeling of the importance of the subject alike to the service, the cadets, and the schools, and said he wished to see both modern languages and science duly encouraged; he thought they might both be made compulsory, as he believed that early education should rest on a wide basis, and that specialising should only be encouraged later. Alluding to the work in science done at the Royal Military Academy, Dr. Percival mentioned that he knew of one cadet who, owing to the absence of any higher teaching there at the earlier stages, was lately learning science which he, the cadet, was well fitted to teach.

The Principal of Cheltenham College, Mr. James, confessed that his own interests and convictions on educational matters were those of a linguist rather than those of a man of science; but practical experience showed him that the present system told most unfairly against scientific boys who entered Woolwich; science was being gradually edged out. Many other head masters of public schools felt with the deputation. He thought also that the present system tended to the disadvantage of the smaller schools, where science was often exceedingly well taught. He hoped that in making any changes the authorities would be careful to consider the interests of linguistic boys, and

would not add to the number of subjects taken up at entrance, for boys were already overburdened in their preparation.

The Head Master of Clifton College, Mr. Glazebrook, said that this was a question on which the public schools had a strong claim to be heard, since an increasing number of boys passed direct from them to Woolwich—the proportion last July being about four-fifths of all the candidates. But the discouragement of science was not so serious to the great schools as to the smaller and less expensive schools, where as a rule science is well taught, but not German. He thought it undesirable that these latter should be debarred from competition. It was not only by the assignment of marks that science was now discouraged, but also by the system of instruction. Boys who went up to Woolwich tolerably proficient in chemistry were put back to the elements, and at the end of their first year knew less than when they entered. Such boys were naturally inclined to complain that science at Woolwich was a farce, and to urge their friends at school to take up another subject which was treated more seriously.

Further remarks were made by Sir B. Samuelson, who especially advocated the encouragement of all types of boys from the public schools, by Prof. Jelf, and by Mr. Shenstone. Statements were made by the Director-General of Military Education and the Inspector-General of Fortifications; the latter officer emphasised the importance of German and of electricity, and said many cadets were markedly deficient in the latter subjects when they left Woolwich. In concluding, Mr. Campbell-Bannerman expressed his obligation to the deputation, and his sense of the importance of the matter brought under his notice, which would have his most careful attention.

It will be seen from this report that the position of cadets of scientific ability at the Royal Military Academy is, as we pointed out some time ago, far from satisfactory, and that this view is now not only held by men of science but also by many head masters and by distinguished members of the military profession, who on this and on other occasions recently have spoken clearly on the subject.

The main defects of the present system seem to be:—(1) That science and German, two subjects which ought to go hand in hand in the early education of officers of the scientific branches, are at present brought into distinct conflict; (2) that in effect so great a bonus is given to German in the course of work at the Royal Military Academy as to be likely very soon to drive science out of the entrance examination, and to a corresponding extent out of the public schools; (3) that the standard of work of the cadets in science, and particularly in electricity when they leave the Royal Military Academy, is lower than it ought to be in very many cases.

Of these defects the last, which is doubtless largely the outcome of the first two, is probably the most important, and it will never be remedied so long as the authorities cling to the idea that a sufficient knowledge of several branches of science can be given to the cadets, even when they are quite new to such studies, in the moderate amount of time that can be spared for them during the comparatively brief course of work at the Royal Military Academy. That this idea is wrong we have pointed out again and again. If those who are responsible for the education of the cadets at Woolwich really desire that the cadets shall attain to a higher standard in science, they must not only encourage the admission of lads of scientific ability, but they must either set apart much more time to such work at the Academy, and give opportunities for, and more encouragement to, advanced work on the part of those who take up the subject, and do well in it at the entrance examination; or, if the giving of more time to science at the Royal Military Academy is impracticable, as is very possibly the case, they must so alter the conditions of the entrance examination as to secure that the cadets shall learn their elementary chemistry and heat at school, and be able to devote their science work at Woolwich wholly to electricity, which is technically of such great importance to

them, but to which at present they can only give a portion of their time.

By doing this the authorities of the Academy will not only advance the interests of the service, they will also avoid that discouragement of the more scientific cadets and of the teaching of science in schools which is admittedly a result of the present system as a whole.

In conclusion, we would urge strongly what was pointed out by Sir Henry Roscoe on Friday, that it is not merely scientific knowledge but scientific ability which is wanted, and that it is only by giving due weight to science at the entrance examination and afterwards that this can be secured.

#### CLIMBING PLANTS.<sup>1</sup>

THIS forms the fourth part of A. F. W. Schimper's "Botanische Mittheilungen aus den Tropen," and is devoted to the description and illustration of the various adaptations for climbing exhibited by native Brazilian plants observed on the spot. Following Darwin, the author distinguishes four different classes of climbing plants, according to the manner in which they climb; but his four classes are not quite the same. Darwin divided them into those having stems which twine spirally round a support; those which climb by means of irritable organs; those which climb by means of hooks; and those which climb by means of roots. Darwin's investigations, it will be remembered, were chiefly directed to the elucidation of the phenomena exhibited by twiners, and such plants as climb by means of tendrils. Schenck treats in a general way of all four classes of climbers; and his work is more in the nature of a text-book than an account of experimental research. He divides climbing plants into Spreizklimmer, Wurzelkletterer, Windepflanzen, and Rankenpflanzen, corresponding nearly to the hook, root, twining, and tendril climbers of Darwin and others. But the Spreizklimmer include all climbing plants that neither twine nor possess either irritable climbing organs or clinging roots, whether armed or unarmed. Thus the least organised of climbing plants are those having weak, slender, rampant stems and branches which grow up among other plants and rest upon them without any other means of support; whilst the most perfectly developed climbing plants are those provided with highly sensitive nutating tendrils, such as the Cucurbitaceæ and the Passifloraceæ. It is difficult to find an exact English equivalent for "Spreizklimmer," but "incumbent climbers" might be employed to designate this class. Twiners revolve with the sun, as the hop (*Humulus Lupulus*), or against the sun, as the scarlet-runner bean (*Phaseolus vulgaris*); but Schenck agrees with Darwin and other observers that they are not sensible to contact. It is only the plants classed as tendril-climbers that exhibit this property; and this irritability is developed both in caulomes and in phyllomes—that is in branches and in leaves, more or less modified for the purpose. In England there are only three woody climbers, namely: the ivy, a root-climber; the honeysuckle, a twiner; and *Clematis vitalba*, a leaf-stalk climber; but in Brazil, and in other tropical countries, they are exceedingly numerous, and present a great variety of adaptations to this end. Dr. Schenck, however, does not confine himself to Brazilian forms. He briefly reviews all the types that have come under his observation. Plants climbing by means of tendrils (irritable organs), conceived in the widest sense, are classified according to the organs, or parts of the organs, by means of which they climb. First he takes the leaf-climbers, which climb by means of

<sup>1</sup> "Beiträge zur Biologie und Anatomie der Lianen im Besonderen der in Brasilien einheimischen Arten." Mit 7 Tafeln. Von Dr. H. Schenck. (Jena: Gustav Fischer, 1892.)